CLASS X (2019-20)
MATHEMATICS STANDARD(041)
SAMPLE PAPER-7

Time : 3 Hours **Maximum Marks : 80**

General Instructions :

(i) All questions are compulsory.
(ii) The questions paper consists of 40 questions divided into 4 sections A, B, C and D.
(iii) Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 8 questions of 3 marks each. Section D comprises of 6 questions of 4 marks each.
(iv) There is no overall choice. However, an internal choices have been provided in two questions of 1 mark each, two questions of 2 marks each, three questions of 3 marks each, and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.
(v) Use of calculators is not permitted.

Section A

Q.1-Q.10 are multiple choice questions. Select the most appropriate answer from the given options.

1. The value of $(12)^{3^x}+(18)^{3^x}$, $x\in N$, end with the digit. [1]
(a) 2
(b) 8
(c) 0
(d) Cannot be determined

Ans : (c) 0

For all $x\in N$, $(12)^{3^x}$ ends with either 8 or 2 and $(18)^{3^x}$ ends with either 2 or 8.
If $(12)^{3^x}$ ends with 8, then $(18)^{3^x}$ ends with 2.
If $(12)^{3^x}$ ends with 2, then $(18)^{3^x}$ ends with 8.
Thus, $(12)^{3^x}+(18)^{3^x}$ ends with 0 only.

2. On dividing x^3-3x^2+x+2 by a polynomial $g(x)$, the quotient and remainder were $x-2$ and $-2x+4$ respectively, then $g(x)$ is equal to [1]
(a) x^2+x+1 (b) x^2+1
(c) x^2-x+1 (d) x^2-1

Ans : (c) x^2-x+1

Here,
$$\text{Dividend} = x^3-3x^2+x+2$$
$$\text{Quotient} = x-2$$
$$\text{Remainder} = -2x+4 \text{ and}$$
$$\text{Divisor} = g(x)$$

Since,
$$\text{dividend} = \text{Divisor}\times\text{Quotient} + \text{Remainder}$$

So,
$$x^3-3x^2+x+2 = g(x)\times(x-2)+(-2x+4)$$
$$g(x)\times(x-2) = x^3-3x^2+x+2+2x-4$$
$$g(x) = \frac{x^3-3x^2+3x-2}{x-2}$$
$$= \frac{(x-2)(x^2-x+1)}{(x-2)}$$
$$= x^2-x+1$$

3. At present ages of a father and his son are in the ratio $7:3$, and they will be in the ratio 2 : 1 after 10 years. Then the present age of father (in years) is [1]
(a) 42 (b) 56
(c) 70 (d) 77

Ans : (c) 70

Let the ages of father and son be $7x$, $3x$

Hence,
$$(7x+10):(3x+10) = 2:1$$
$$7x+10 = 6x+20$$
$$7x-6x = 20-10$$
or
$$x = 10$$

Age of the father is 70 years.

4. Each root of $x^2-bx+c=0$ is decreased by 2. The resulting equation is $x^2-2x+1=0$, then [1]
(a) $b=6, c=9$ (b) $b=3, c=5$
(c) $b=2, c=-1$ (d) $b=-4, c=3$

Ans : (a) $b=6, c=9$

$$\alpha+\beta = b$$
$$\alpha\beta = c$$

According to the question
$$(\alpha+\beta-4) = b-4$$
$$(\alpha-2)(\beta-2) = \alpha\beta-2(\alpha+\beta)+4$$
$$= c-2b+4$$

Now
$$2 = b-4$$
$$b = 6$$
$$1 = c-2b+4$$
$$1 = c-2\times 6+4 = c-12+4$$
$$c = 1+12-4 = 9$$

5. What is the common difference of four terms in A.P. such that the ratio of the product of the first fourth term to that of the second and third term is $2:3$ and the sum of all four terms is 20? [1]
(a) 3 (b) 1
(c) 4 (d) 2

Ans : (d) 2

Take the four terms as $a-3x$, $a-x$, $a+x$, $a+3x$
$$\text{The sum} = 4a = 20$$
$$a = 5$$

Also, $3(a^2-(3x)^2) = 2(a^2-x^2)$

$x = 1$

However, the common difference is $2x$ and not x

When, $x = 1,\ d = 2x = 2$

6. The ratio in which the point $(2, y)$ divides the join of $(-4, 3)$ and $(6, 3)$. The value of y is [1]

(a) $2:3, y=3$ (b) $3:2, y=4$

(c) $3:2, y=3$ (d) $3:2, y=2$

Ans : (c) $3:2, y=3$

Let the required ratio be $k:1$

Then, $2 = \frac{6k-4(1)}{k+1}$

or $k = \frac{3}{2}$

The required ratio is $\frac{3}{2}:1$ or 3 : 2

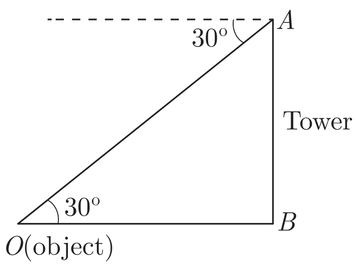
Also, $y = \frac{3(3)+2(3)}{3+2} = 3$

7. If the angle of depression of an object from a 75 m high tower is 30°, then the distance of the object from the tower is [1]

(a) $25\sqrt{3}$ m (b) $50\sqrt{3}$ m

(c) $75\sqrt{3}$ m (d) 150 m

Ans : (c) $75\sqrt{3}$ m



$$\tan 30^\circ = \frac{AB}{OB}$$

$$\frac{1}{\sqrt{3}} = \frac{75}{OB}$$

$$OB = 75\sqrt{3} \text{ m}$$

8. Ratio of volumes of two cones with same radii is [1]

(a) $h_1:h_2$ (b) $s_1:s_2$

(c) $r_1:r_2$ (d) None of these

Ans : (a) $h_1:h_2$

$$\frac{1}{3}\pi r_1^2 h_1 : \frac{1}{3}\pi r_2^2 h_2$$

$$\frac{1}{3}\pi r_1^2 h_1 : \frac{1}{3}\pi r_1^2 h_2 \qquad (r_1 = r_2)$$

$$h_1 : h_2$$

9. In a frequency distribution, the mid value of a class is 10 and the width of the class is 6. The lower limit of the class is [1]

(a) 6 (b) 7

(c) 8 (d) 12

Ans : (b) 7

Let x be the upper limit and y be the lower limit. Since the mid value of the class is 10.

Hence, $\frac{x+y}{2} = 10$

$x + y = 20$ ...(1)

and $x - y = 6$ (width of the class = 6)..(2)

By solving (1) and (2), we get $y = 7$

Hence, lower limit of the class is 7.

10. The probability of getting a number greater than 2 in throwing a dice is [1]

(a) 2/3 (b) 1/3

(c) 4/3 (d) 1/4

Ans : (a) 2/3

Required probability $= \frac{4}{6} = \frac{2}{3}$

(Q.11-Q.15) Fill in the blanks.

11. The ratio of the areas of two similar triangles is equal to the square of the ratio of their .... [1]

Ans : corresponding sides

12. Point $(-4, 6)$ divide the line segment joining the points $A(-6, 10)$ and $B(3, -8)$ in the ratio ....[1]

Ans : 2 : 7

or

All the points equidistant from two given points A and B lie on the .... of the line segment AB.

Ans : perpendicular bisector

13. It $\tan A = 4/3$ then $\sin A$ .... [1]

Ans : 4/5

14. A line that intersects a circle in one point only is called .... [1]

Ans : tangent

15. Two points on a line segment are marked such that the three parts they make are equal then we say that the two points .... the line segment. [1]

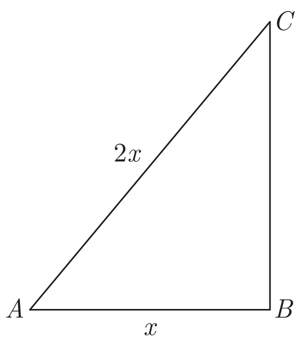
Ans : Trisect

(Q.16-Q.20) Answer the following

16. If the length of the ladder placed against a wall is twice the distance between the foot of the ladder and the wall. Find the angle made by the ladder with the horizontal. [1]

Ans :

Let the distance between the foot of the ladder and the wall is x, then length of the ladder will be $2x$. As per given in question we have drawn figure below.



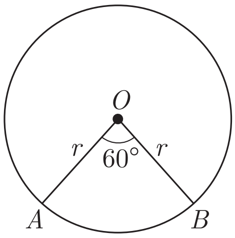
In ΔABC, $\angle B = 90^\circ$

$$\cos A = \frac{x}{2x}$$
$$= \frac{1}{2} = \cos 60^{\circ}$$
$$A = 60^{\circ}$$

17. What is the perimeter of the sector with radius 10.5 cm and sector angle 60º. [1]

Ans :

As per question the digram is shown below.



Perimeter of the sector,

$$p = 2r + \frac{2\pi r\theta}{360^{\circ}}$$
$$= 10.5 \times 2 + 2 \times \frac{22}{7} \times \frac{10.5 \times 60}{360}$$
$$= 21 + 11 = 32 \text{ cm}$$

18. Two cubes each of volume 8 cm^3 are joined end to end, then what is the surface area of resulting cuboid. [1]

Ans :

Given

Side of the cube, $a = \sqrt[3]{8} = 2$ cm

Now the length of cuboid

$$l = 4 \text{ cm}$$

Breadth, $b = 2$ cm

Height, $h = 2$ cm

Surface area of cuboid

$$= 2(l \times b + b \times h + h \times l)$$
$$= 2(4 \times 2 + 2 \times 2 + 2 \times 4)$$
$$= 2 \times 20 = 40 \text{ cm}^2$$

or

A solid metallic object is shaped like a double cone as shown in figure. Radius of base of both cones is same but their heights are different. If this cone is immersed in water, find the quantity of water it will displace.

Ans :

$$\text{Volume of the upper cone} = \frac{1}{3}\pi r^2 h$$
$$\text{Volume of the lower cone} = \frac{1}{3}\pi r^2 H$$
$$\text{Total volume of both the cones} = \frac{1}{3}\pi r^2 h + \frac{1}{3}\pi r^2 H$$
$$= \frac{1}{3}\pi r^2 (h + H)$$

The quantity of water displaced will $\frac{1}{3}\pi r^2(h + H)$ cube units.

19. Find the following frequency distribution, find the median class : [1]

Cost of living index	1400-1500	1550-1700	1700-1850	1850-2000
Number of weeks	8	15	21	8

Ans :

C.I.	1400-1550	1550-1700	1700-1850	150-2000
f	8	15	21	8
c.f.	8	23	44	52

$$\frac{\Sigma f}{2} = 26 \Rightarrow \text{Median class} = 1700 - 1850$$

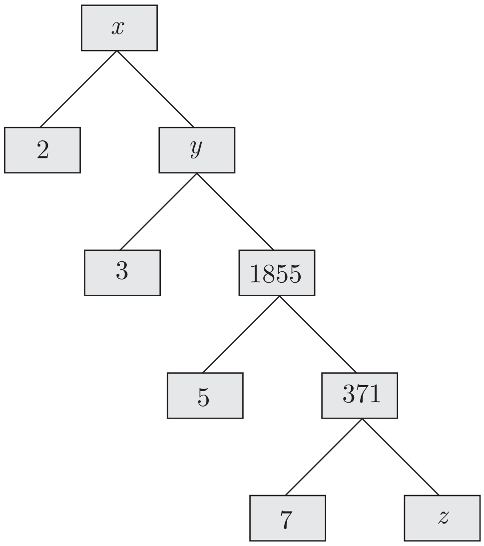
20. Out of 200 bulbs in a box, 12 bulbs are defective. One bulb is taken out at random from the box. What is the probability that the drawn bulb is not defective? [1]

Ans :

$$\text{Total No. of cases} = 200$$
$$\text{Favourable cases} = 200 - 12$$
$$= 188$$
$$\text{Required probability} = \frac{188}{200}$$
$$= \frac{47}{50}$$

Section B

21. Complete the following factor tree and find the composite number x [2]

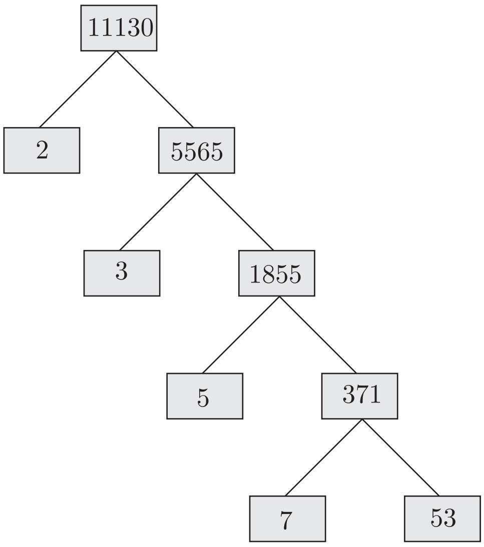


Ans :

We have $z = \frac{371}{7} = 53$

$$y = 1855 \times 3 = 5565$$
$$x = 2 \times y = 2 \times 5565 = 11130$$

Thus complete factor three is as given below.



22. If $x = -\frac{1}{2}$, is a solution of the quadratic equation $3x^2 + 2kx - 3 = 0$, find the value of k. [2]

Ans :

We have $\quad 3x^2 + 2kx - 3 = 0$

Putting $x = -\frac{1}{2}$, we get

$$3\left(-\frac{1}{2}\right)^2 + 2k\left(-\frac{1}{2}\right) - 3 = 0$$

$$\frac{3}{4} - k - 3 = 0$$

$$k = \frac{3}{4} - 3$$

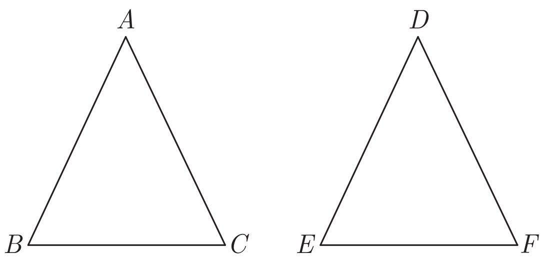
$$= \frac{3-12}{4} = \frac{-9}{4}$$

Hence $k = \frac{-9}{4}$

23. The sides AB and AC and the perimeter P_1 of ΔABC are respectively three times the corresponding sides DE and DF and the parameter P_2 of ΔDEF. Are the two triangles similar? If yes, find $\frac{ar(\Delta ABC)}{ar(\Delta DEF)}$ [2]

Ans :

As per given condition we have drawn the figure below.



In ΔABC and ΔDEF,

$$AB = 3DE$$

and $\quad AC = 3DF$

$$\frac{AB}{DE} = 3;\ \frac{AC}{DF} = 3;$$

Since $P_1 = 3P_2$, $\ BC = 3EF$

Thus $\quad \frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF} = 3$

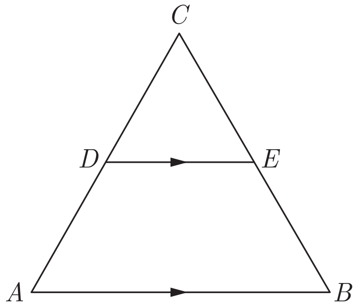
By SSS criterion we have

$$\Delta ABC \sim \Delta DEF$$

$$\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \left(\frac{AB}{DE}\right)^2 = (3)^2 = 9$$

or

In the given figure, $\angle A = \angle B$ and $AD = BE$. Show that $DE || AB$.



Ans :

In ΔCAB, we have

$$\angle A = \angle B \quad (1)$$

By isoscales triangle property, we have

$$AC = CB$$

But, we have been given

$$AD = BE \quad (2)$$

Dividing equation (2) by (1) we get,

$$\frac{CD}{AD} = \frac{CE}{BE}$$

By converse of BPT,

$DE || AB$. Hence Proved

24. Two slips of paper marked 5 and 10 are put in a box and three slips marked 1, 3, 5 are in another. One slip from each box is drawn. [2]

(a) What is the probability that both show odd number?

(b) What is the probability of getting one odd number and one even number?

Ans :

One box contains (5,10)

Other box contains (1, 3, 5)

(a) For I box probability for odd number $\frac{1}{2}$

Fro II box probability for odd number $= \frac{3}{3} = 1$

Required probability $= \frac{1}{2} \times 1 = \frac{1}{2}$

(b) P (one odd and one even)

$= P$ (one odd from box I)

$\times P$ (one even from box II)

$+ P$ (one even from box I)

$\times P$ (one odd from box II)

$$= \frac{1}{2} \times \frac{0}{3} + \frac{1}{2} \times \frac{3}{3}$$

$$= 0 + \frac{1}{2} = \frac{1}{2}$$

25. The data regarding marks obtained by 48 students of a class in a class test is given below. Calculate the modal marks of students. [2]

Marks obtained	0-5	5-10	10-15	15-20	20-25	25-30	30-35	35-40	40-45	45-50
Number of students	1	0	2	0	0	10	25	7	2	1

Ans :

Modal class is 30-35, $l = 30$, $f_1 = 25$ $f_0 = 10$, $f_2 = 7$, $h = 5$

$$\text{Mode } = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$\Rightarrow \quad \text{Mode } = 30 + \frac{25 - 10}{50 - 10 - 7} \times 5$$

$$= 30 + 2.27 \text{ or } 32.27 \text{ approx.}$$

or

The following table gives the life time in days of 100 bulbs :

Life time in days	Less than 50	Less than 100	Less than 150	Less than 200	Less than 250	Less than 300
Number of Bulbs	8	23	55	81	93	100

Change the above distribution as frequency distribution.

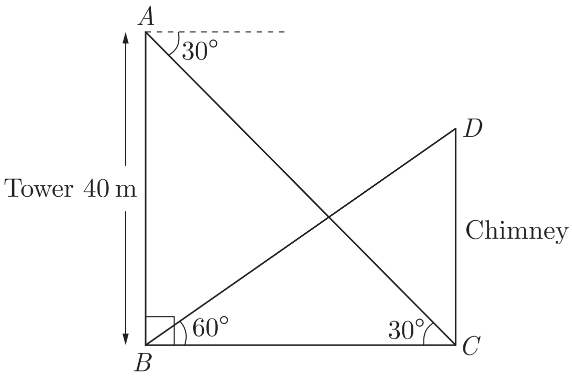
Ans :

Frequency distribution table.

Class -Interval	Frequency
0-50	8
50-100	15
100-150	32
150-200	26
200-250	12
250-300	7
Total	100

26. The angle of elevation of the top of a chimney from the foot of a tower is 60° and the angle of depression of the foot of the chimney from the top of the tower is 30°. If the height of tower is 40 m, find the height of smoke emitting chimney. According to pollution control norms, the minimum height of a smoke emitting chimney should be 100 m. What value is discussed in this problem? [2]

Ans :



Given $AB = 40$ m be the height of the tower and CD be the height of smoking chimney.
In right ΔABC,

$$\tan 30^\circ = \frac{AB}{BC} \Rightarrow \frac{1}{\sqrt{3}} = \frac{40}{BC}$$

$$BC = 40\sqrt{3}$$

Again, in right ΔDCB,

$$\tan 60^\circ = \frac{DC}{BC}$$

$$\sqrt{3} = \frac{DC}{40\sqrt{3}}$$

$$DC = 120 \text{ m}$$

The height of chimney is 100 m,
Which is greater than the ideal height 100 m of a small emitting chimney.

Section C

27. Find the HCF of 180, 252 and 324 by Euclid's Division algorithm. [3]

Ans :

We have $324 = 252\times1 + 72$

$252 = 72\times3 + 36$

$72 = 36\times2 + 0$

Thus $HCF(324, 252) = 36$

Now $180 = 36\times5 + 0$

Thus $HCF(36, 180) = 36$

Thus HCF of 180, 252, and 324 is 36.
Hence required number $= 999999 - 63 = 999936$

or

144 cartons of Coke cans and 90 cartons of Pepsi cans are to be stacked in a canteen. If each stack is of the same height and if it equal contain cartons of the same drink, what would be the greatest number of cartons each stack would have?

Ans :

The required answer will be HCF of 144 and 90.

$$144 = 2^4\times3^2$$

$$90 = 2\times3^2\times5$$

$$\text{HCF}(144, 90) = 2\times3^2 = 18$$

Thus each stack would have 18 cartons.

28. Solve for x : [3]

$$\frac{x+1}{x-1} + \frac{x-2}{x+2} = 4 - \frac{2x+3}{x-2}; x \neq 1, -2, 2$$

Ans :

We have $$\frac{x+1}{x-1} + \frac{x-2}{x+2} = 4 - \frac{2x+3}{x-2}$$

$$\frac{x^2+3x+2+x^2-3x+2}{x^2+x-2} = \frac{4x-8-2x-3}{x-2}$$

$$\frac{2x^2+4}{x^2+x-2} = \frac{2x-11}{x-2}$$

$$(2x^2+4)(x-2) = (2x-11)(x^2+x-2)$$

$$5x^2+19x-30 = 0$$

$$(5x-6)(x+5) = 0$$

$$x = -5, \frac{6}{5}$$

29. The ninth term of an A.P. is equal to seven times the second term and twelfth term exceeds five times the third term by 2. Find the first term and the common difference. [3]

Ans :

Let the first term be a, common difference be d and nth term be a_n.

Now $a_9 = 7a_2$

$$a+8d = 7(a+d)$$

$$a+8d = 7a+7d$$

$$-6a+d = 0 \quad (1)$$

and $a_{12} = 5a_3+2$

$$a+11d = 5(a+2d)+2$$

$$a+11d = 5a+10d+2$$

$$-4a+d = 2 \quad ...(2)$$

Subtracting (2) from (1), we get

$$-2a = -2$$

$$a = 1$$

Substituting this value of a in (1) we get

$$-6+d = 0$$

$$d = 6$$

Hence first term is 1 and common difference is 6.

or

Find the 20^{th} term of an A.P. whose 3^{rd} term is 7 and the seventh term exceeds three times the 3^{rd} term by 2. Also find its n^{th} term (a_n).

Ans :

Let the first term be a, common difference be d and nth term be a_n.

We have $a_3 = a+2d = 7 \quad (1)$

$$a_7 = 3a_3+2$$

$$a+6d = 3\times 7+2 = 23 \quad (2)$$

Solving (1) and (2) we have

$$4d = 16 \Rightarrow d = 4$$

$$a+8 = 7 \Rightarrow a = -1$$

$$a_{20} = a+19d = -1+19\times 4 = 75$$

$$a_1 = a+(n-1)d = -1+4n-4$$

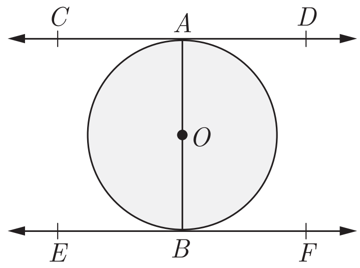
$$= 4n-5.$$

Hence n^{th} term is $4n-5$

30. Prove that the tangents drawn at the ends of a diameter of a circle are parallel. [3]

Ans :

Let AB be a diameter of a given circle and let CD and RF be the tangents drawn to the circle at A and B respectively as shown in figure below.



Here $AB \perp CD$ and $AB \perp EF$

Thus $\angle CAB = 90^\circ$ and $\angle ABF = 90^\circ$

Hence $\angle CAB = \angle ABF$

and $\angle ABE = \angle BAD$

Hence $\angle CAB$ and $\angle ABF$ also $\angle ABE$ and $\angle BAD$ are alternate interior angles.

$CD || EF$ Hence Proved

31. Read the following, understand the mathematical idea expressed in it answer the questions that follow: 1,4,9,16, ... are the square of the counting numbers. The remainders got by dividing the square numbers with natural numbers have a cyclic property. For example, the remainders on dividing these numbers by 4 are tabulated here. [3]

Number	1	4	9	16	25	-	-	-
Remainder	1	0	1	0	1	-	-	-

On dividing by 4 perfect squares leave only 0 and 1 as remainders. From this we can conclude that an arithmetic sequence whose terms leaves remainder 2 on dividing by 4 do not have a perfect square.

(a) Which are the possible remainders on dividing any number with 4?

(b) Which are the numbers we would not get on dividing a perfect square by 4?

(c) What is the remainder that leaves on dividing the terms of the arithmetic sequence 2,5,8,11, ... by 4?

Ans :

(a) Any number can be form of $(4d+r)$
Where $r = 0,1,2$ and 3
When any number divided by 4 remainders are 0, 1, 2, and 3.

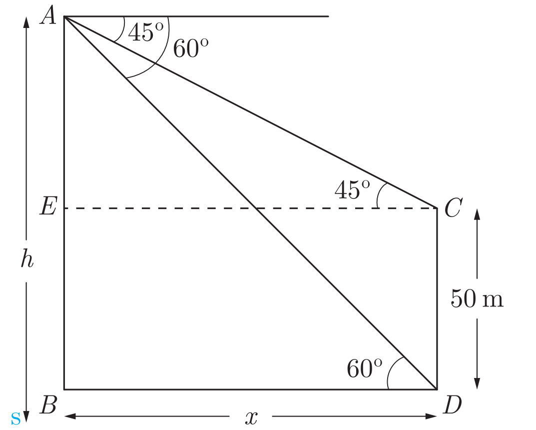
(b) A perfect square number divided by 4 leave the remainder 0 and 1
2 and 3 are not get as remainder when perfect square number divided by 4.

(c) 2, 5, 8, 11 ......
remainders are 2, 1, 0, 3 ....

32. The angles of depression of the top and bottom of a 50 m high building from the top of a tower are 45° and 60° respectively. Find the height of the tower and the horizontal distance between the tower and the building. (Use $\sqrt{3} = 1.73$) [3]

Ans :

As per given in question we have drawn figure below.



We have $\tan 45^{\circ} = \frac{h-50}{x}$

$$x = h - 50 \qquad ...(1)$$

and $\tan 60^{\circ} = \frac{h}{x}$

$$\sqrt{3} = \frac{h}{x}$$

$$x = \frac{h}{\sqrt{3}} \qquad ...(2)$$

From (1) and (2) we have

$$h - 50 = \frac{h}{\sqrt{3}}$$

$$\sqrt{3}\,h - 50\sqrt{3} = h$$

$$\sqrt{3}\,h - h = 50\sqrt{3}$$

$$h(\sqrt{3}-1) = 50\sqrt{3}$$

$$h = \frac{50\sqrt{3}}{\sqrt{3}-1} = \frac{50(3+\sqrt{3})}{2}$$

$$h = 25(3+\sqrt{3}) = 75 + 25\sqrt{3}$$

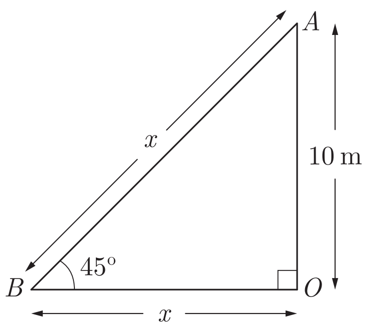
$$= 118.30 \text{ m}$$

or

An electric pole is 10 m high. A steel wire tied to top of the pole is affixed at a point on the ground to keep the pole up right. If the wire makes an angle of 45° with the horizontal through the foot of the pole, find the length of the wire. [Use $\sqrt{2} = 1.414$]

Ans :

Let OA be the electric pole and B be the point on the ground to fix the pole. Let BA be x.
As per given in question we have drawn figure below.



In $\Delta\, ABO$, we have

$$\sin 45^{\circ} = \frac{AO}{AB}$$

$$\frac{1}{\sqrt{2}} = \frac{10}{x}$$

$$x = 10\sqrt{2} = 10 \times 1.414$$

$$= 14.14 \text{ m}$$

Hence, the length of wire is 14.14 m

33. The sum of the radius of base and height of a solid right circular cylinder is 37 cm. If the total surface area of the solid cylinder is 1628 sq. cm, find the volume of the cylinder. $\pi = \frac{22}{7}$ [3]

Ans :

Here, $r + h = 37 \qquad (1)$

and $2\pi r(r+h) = 1628 \qquad (2)$

Thus $2\pi r \times 37 = 1628$

$$2\pi r = \frac{1628}{37}$$

$$r = 7 \text{ cm}$$

Substituting $r = 7$ in (1) we have

$$h = 30 \text{ cm.}$$

Here volume of cylinder

$$\pi r^2 h = \frac{22}{7} \times 7 \times 7 \times 30 = 4620 \text{ cm}^3$$

34. From a solid wooden sphere with 13 centimetres radius, a cone with 18 centimetres height and maximum base is made. [3]

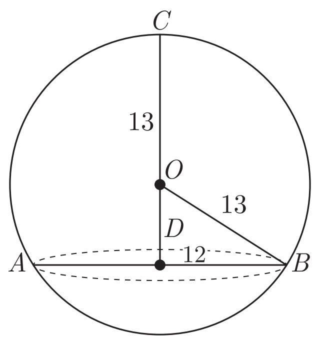
(a) Taking the base radius of the cone as r. draw a rough figure.

(b) Calculate the radius of the cone.

(c) What is the volume of the cone?

Ans :

(a)



(b) $OC = OB = 12$

$$OD = 18 - OC = 18 - 13 = 5$$

$$r = BD = 12 \text{ cm}$$

(c) volume of cone $= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 144 \times 18$

$$= 2715.4 \text{ cm}^3$$

Section D

35. If the polynomial $x^4 - 6x^3 + 16x^2 - 25x + 10$ is divided by $(x^2 - 2x + k)$, the remainder comes out to be $x + a$, find k and a. [4]

Ans :

$$\begin{array}{r|l} & x^2-4x+(8-k) \\ x^2-2x+k & x^4-6x^3+16x^2-25x+10 \\ & x^4-2x^3+kx^2 \\ \hline & -4x^3+(16-k)x^2-25x+10 \\ & -4x^3 \quad +8x^2-4kx \\ \hline & (8-k)x^2-(25-4k)x+10 \\ & (8-k)x-(16-2k)x+(8k-k^2) \\ \hline & (2k-9)x+(10-8k+k^2) \end{array}$$

Given, remainder $= x + a$

Comparing the multiples of x

$$(2k-9)x = 1 \times x$$

$$2k - 9 = 1$$

$$k = \frac{10}{2} = 5$$

Substituting this value of k into other portion of remainder, we get

and $\quad a = 10 - 8k + k^2 = 10 - 40 + 25$

$$= -5$$

or

Obtain all other zeroes of the polynomial $9x^4 - 6x^3 - 35x^2 + 24x - 4$, if two of its zeroes are 2 and -2.

Ans :

As 2 and -2 are the zeroes of $9x^4 - 6x^3 - 35x^2 + 24x - 4$, So $(x-2)$ and $(x+2)$ are its two factors and

$$(x-2)(x+2) = x^2 - 4$$

Dividing $9x^4 - 6x^3 - 35x^2 + 24x - 4$ by $x^2 - 4$

$$\begin{array}{r|l} & 9x^2-6x+1 \\ x^2-4 & 9x^4-6x^3-35x^2+24x-4 \\ & 9x^4 \quad -36x^2 \\ \hline & -6x^3+x^2+24x-4 \\ & -6x^3 \quad +24x \\ \hline & x^2 \quad -4 \\ & x^2 \quad -4 \\ \hline & 0 \end{array}$$

Factorising this quotient

$$= [9x^2 - 3x - 3x + 1]$$

$$= [3x(3x-1) - 1(3x-1)]$$

$$= [(3x-1)(3x-1)]$$

$$= (3x-1)(3x-1)$$

Hence, other two zeroes are $\frac{1}{3}, \frac{1}{3}$.

36. Solve the following pair of equations : [4]

$$\frac{2}{\sqrt{x}} + \frac{3}{\sqrt{y}} = 2 \text{ and } \frac{4}{\sqrt{x}} - \frac{9}{\sqrt{y}} = -1$$

Ans :

We have $\quad \frac{2}{\sqrt{x}} + \frac{3}{\sqrt{y}} = 2 \quad \frac{4}{\sqrt{x}} - \frac{9}{\sqrt{y}} = -1$

Substitute $\frac{1}{\sqrt{x}} = X$ and $\frac{1}{\sqrt{y}} = Y$

$$2X + 3Y = 2 \qquad \text{...(1)}$$

$$4X - 9Y = -1 \qquad \text{...(2)}$$

Multiplying equation (1) by 3, and adding in (2) we get

$$10X = 5 \Rightarrow X = \frac{5}{10} = \frac{1}{2}$$

Thus $\quad \frac{1}{\sqrt{x}} = \frac{1}{2} \Rightarrow x = 4$

Putting the value of X in equation (1), we get

$$2 \times \frac{1}{2} + 3y = 2$$

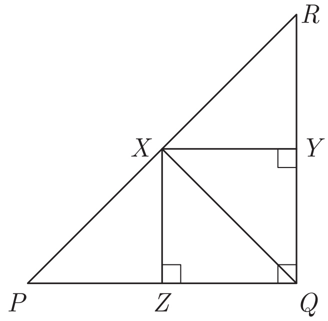
$$3Y = 2 - 1$$

$$Y = \frac{1}{3}$$

Now $\quad Y = \frac{1}{3} \Rightarrow \frac{1}{\sqrt{y}} = \frac{1}{3} \Rightarrow y = 9$

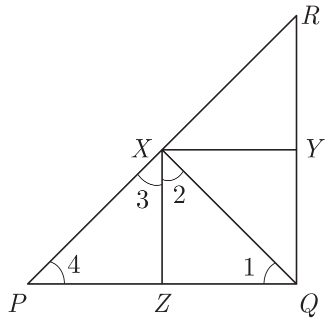
Hence $x = 4$, $y = 9$.

37. ΔPQR is right angled at Q, $QX \perp PR$, $XY \perp RQ$ and $XZ \perp PQ$ are drawn. Prove that $XZ^2 = PZ \times ZQ$. [4]



Ans :

We have redrawn the given figure as below.



It may be easily seen that $\quad RQ \perp PQ$

and $XZ \perp PQ$ or $XZ \parallel YQ$

Similarly $\quad XY \parallel ZQ$

Thus $XYQZ$ is a rectangle.

In ΔXZQ, $\quad \angle 1 + \angle 2 = 90^\circ \qquad$ ...(1)

and in ΔPZX, $\quad \angle 3 + \angle 4 = 90^\circ \qquad$ ...(2)

$XQ \perp PR$ or, $\quad \angle 2 + \angle 3 = 90^\circ \qquad$ ...(3)

From eq. (1) and (3), $\quad \angle 1 = \angle 3$

From eq. (2) and (3), $\quad \angle 2 = \angle 4$

Due to AA similarity

$$\Delta PZX \sim \Delta XZQ$$

$$\frac{PZ}{XZ} = \frac{XZ}{ZQ}$$

$$XZ^2 = PZ \times ZQ$$

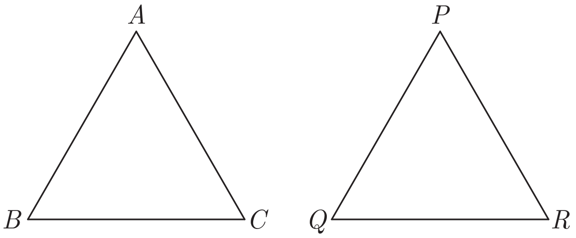
Hence proved

or

If the area of two similar triangles are equal, prove that they are congruent.

Ans :

As per given condition we have drawn the figure below.



We have $\Delta ABC \sim \Delta PQR$,

and $ar\Delta ABC = ar\Delta PQR$

Since $\Delta ABC \sim \Delta PQR$, we have

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{CA^2}{RP^2} \quad ...(1)$$

Since $ar(\Delta ABC) = ar(\Delta PQR)$ we have

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = 1$$

From equation (1), we get

$$\frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{CA^2}{RP^2} = 1$$

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP} = 1$$

$$AB = PQ,$$

$$BC = QR$$

and $CA = RA$

By SSS similarity we have

$$\Delta ABC \cong \Delta PQR$$

38. Evaluate : [4]

$\tan^2 30^\circ \sin 30^\circ + \cos 60^\circ \sin^2 90^\circ \tan^2 60^\circ - 2\tan 45^\circ \cos^2 0^\circ \sin 90^\circ$

Ans :

$\tan^2 30^\circ \sin 30^\circ + \cos 60^\circ \sin^2 90^\circ \tan^2 60^\circ - 2\tan 45^\circ \cos^2 0^\circ \sin 90^\circ$

$$= \left(\frac{1}{\sqrt{3}}\right)^2 \times \frac{1}{2} + \frac{1}{2} \times (1)^2 \times (\sqrt{3})^2 - 2 \times 1 \times 1^2 \times 1$$

$$= \frac{1}{3} \times \frac{1}{2} + \frac{1}{2} \times 1 \times 3 \times - 2 \times 1 \times 1 \times 1$$

$$= \frac{1}{6} + \frac{3}{2} - 2 = \frac{1 + 9 - 12}{6} = -\frac{2}{6} = -\frac{1}{3}$$

or

If $\sqrt{3}\cot^2\theta - 4\cot\theta + \sqrt{3} = 0$, then find the value of $\cot^2\theta + \tan^2\theta$.

Ans :

We have $\sqrt{3}\cot^2\theta - 4\cot\theta + \sqrt{3} = 0$

Let $\cot\theta = x$, then we have

$$\sqrt{3}x^2 - 4x + \sqrt{3} = 0$$

$$\sqrt{3}x^2 - 3x - x + \sqrt{3} = 0$$

$$\sqrt{3}x(x - \sqrt{3}) - 1(x - \sqrt{3}) = 0$$

$$(x - \sqrt{3})(\sqrt{3x} - 1) = 0$$

Thus $x = \sqrt{3}$ or $\frac{1}{\sqrt{3}}$

or $\cot\theta = \sqrt{3}$ or $\cot\theta = \frac{1}{\sqrt{3}}$

Therefore $\theta = 30^\circ$ or $\theta = 60^\circ$

If $\theta = 30^\circ$, then

$$\cot^2 30^\circ + \tan^2 30^\circ = (\sqrt{3})^2 + \left(\frac{1}{\sqrt{3}}\right)^2$$

$$= 3 + \frac{1}{3} = \frac{10}{3}$$

If $\theta = 60^\circ$, then

$$\cot^2 60^\circ + \tan^2 60^\circ = \left(\frac{1}{\sqrt{3}}\right)^2 + (\sqrt{3})^2$$

$$= \frac{1}{3} + 3 = \frac{10}{3}.$$

39. Find the coordinates of the point which divide the line segment joining $A(2, -3)$ and $B(-4, -6)$ into three equal parts. [4]

Ans :

Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ trisect the line joining $A(3, -2)$ and $B(-3, -4)$.

As per question, line diagram is shown below.

P divides AB in the ratio of 1:2 and Q divides AB in the ratio 2:1.

By section formula

$$x_1 = \frac{mx_2 + nx_1}{1+2} \text{ and } y = \frac{my_2 + ny_1}{m+n}$$

$$P(x_1, y_1) = \left(\frac{1(-4) + 2(2)}{2+1}, \frac{2(-6) + 1(-3)}{2+1}\right)$$

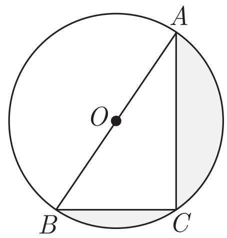
$$= \left(\frac{-4+4}{3}, \frac{-6-(-6)}{3}\right)$$

$$= (0, -4)$$

$$Q(x_2, y_2) = \left(\frac{2(-4) + 1(2)}{2+1}, \frac{2(-6) + 1(-3)}{2+1}\right)$$

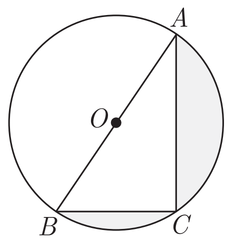
$$= \left(\frac{-8+2}{3}, -\frac{12+(-3)}{3}\right) = (-2, -5)$$

40. In the figure, O is the centre of circle such that diameter $AB = 13$ cm and $AC = 12$cm. BC is joined. Find the area of the shaded region. ($\pi = 3.14$) [4]



Ans :

We redraw the given figure as below.



Radius of semi circle $ACB = \frac{13}{2}$ cm

$$\text{Area of semicircle } = \frac{\pi}{2}r^2 = \frac{3.14}{2}\times\frac{13}{2}\times\frac{13}{2}$$

$$= \frac{3.14\times 169}{8} = \frac{530.66}{8} \text{ cm}^2$$

Semicircle subtend 90° at circle, thus $\angle ACB = 90^\circ$

In ΔABC

$$AC^2 + BC^2 = AB^2$$

$$12^2 + BC^2 = 169$$

$$BC^2 = (160 - 144) = 25$$

$$BC = 5 \text{ cm}$$

Also area $\Delta = \frac{1}{2}\times \text{Base} \times \text{Hight}$

Area of ΔABC $\Delta = \frac{1}{2}\times AC \times BC$

$$= \frac{1}{2}\times 12 \times 5$$

$$= 30 \text{ cm}^2$$

Area of shaded region$= \frac{530.66}{8} - 30$

$$= (66.3325 - 30) \text{ cm}^2$$

$$= 36.3325 \text{ cm}^2$$